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A Decision Theory Model for Ballooning Problems

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Abstract

A balloon flight model constructed to show probability of success at various stages of a flight operation can help the planner make choices between possible courses of action. The value of such a model depends on how well it approximates a real operation and how accurately appropriate probabilities are known. These probabilities include balloon and hardware reliability and the probability that the crew can accomplish each step of the operation successfully. A first model is usually rather crude, and initial probabilities are frequently guesses. With use, the model can be refined and the probabilities needed can be estimated quantitatively through observation. Even the most refined model is only an aid to the man who must make a decision, however. To use it effectively he must understand its nature and supplement it with value judgments made independently of the model. To say this another way, blind use of an excellent decision model may not result in a good decision, but a good model used with understanding can be a valuable tool to the decision maker.

A first model for scientific ballooning is described and used in an example, and a method of getting the necessary reliability and operational probability constants is explained.

21.1 DECISION THEORY

Although the purpose of this paper is to show an application of decision theory to a ballooning problem, a little background in the theory of decision making is necessary to set the stage for understanding the application. First, what is a decision? Strangely, most books and papers on decision theory do not give a specific, concise definition. Essentially, however, the decision process involves recognizing that an action must be taken, seeking out and comparing the consequences of taking the action through each of the various alternative ways open and resolving which of the alternatives to follow. When an alternative has been chosen, a decision has been made. Whether one considers the decision to be the whole process or the resolution made at the end of the process isn't important in the application of decision theory, but it does affect the language one uses in discussing it. I shall use the word "decision" to include the entire process. For a more detailed discussion of the theory and terminology of decision theory, the reader is referred to Eilon (1969), Sisson et al (1967a and 1967b), Fishburn (1964), Wheeler and Peeples (1969), and Tribus (1969).

Decisions can be made through the use of formal procedures, or they may be made quite informally. They may be rational or irrational. They may be good or bad. These words, formal, informal, rational, irrational, good and bad also need to be defined.

A decision was a good one from a decision maker's point of view if the results flowing from the course of action taken following the decision are more favorable to him than the results he believes would have followed other decisions. A decision was bad if he believes another possible decision would have brought more favorable results. Whether a decision was good or bad can not be determined until the consequences of the decision are clear, and even then it is often difficult to weigh those consequences against the possible consequences of other decisions. Goodness or badness is usually relative. In spite of the difficulty of evaluating decisions, it is important in evolving better decision models that an effort be made to do so. Rational, as used here, has a rather special meaning. A rational decision is one which is made by following an agreed-upon decision process and using agreed-upon criteria to specify how a choice between alternatives is to be made. Thus the engineering department of Balloons Ltd., a balloon manufacturer, may have a formal procedure (a Computer Program called BALSPEC-1) for determining the engineering specifications of zero-pressure balloons to meet stated flight requirements. Clearly, such specifications may be arrived at in many other ways, but if company policy dictates the use of BALSPEC-1, it is irrational from the point of view of the company for a company engineer to determine them in any other way.

If one of the Balloons Ltd. engineers, using another computer program, can write equally good (from his point of view) though not identical specifications at half the cost of using BALSPEC-1, his decision on what the balloon specifications should be to meet the requirements will be rational to him but not to the company. On the other hand, if he can convince the company that his specifications are as good as those turned out by BALSPEC-1, his program is likely to be adopted and named BALSPEC-2. Then it would become irrational from both his point of view and the company's to use BALSPEC-1.

The example used above will be used further to illustrate the difference between a formal and an informal decision procedure. When flight requirements are received by Balloons Ltd., they have to be turned into balloon specifications.

Once - say, during the Siege of Paris in 1870, when the Parisians were building Montgolfiers in their railway stations in an effort to communicate with the outside - decisions about balloon design were made very informally. Each designer had his own, rather intuitive ideas about gore patterns. A man, not a machine, decided on the basis of what cloth was available at the moment, etc., how each balloon was to be made. He had many alternatives from which he selected one. He made decisions informally, and most of them were irrational to everyone but him.

Since Balloons Ltd. uses a strictly specified procedure to design balloons, it is formal, but is a decision being made? Are there any choices? If not, no decision was made, because none was necessary. During the evolution of BALSPEC-2, however, decisions were being made. Only after the "mathematical model" of a balloon which was used as a basis for writing BALSPEC-2 became sufficiently realistic, and the criteria for judging what constituted an acceptable balloon were clearly defined, could the procedure become so formal that no alternatives were considered to exist. The decision process had been used in selecting a model and in determining criteria for selecting one among the various alternative materials, etc. The engineers who wrote BALSPEC-1 and 2 had, in their view, solved an engineering problem. So they had! But they had gone through a decision making process in doing it.

Now that I've shown that decision making, even formal decision making, is not new to anyone involved in scientific ballooning, I want to construct a mathematical model of a different aspect of scientific ballooning. My mathematical model will be a version of the "decision tree" which is currently in vogue among writers on decision making, for example Archibald et al (1967). Rather than discuss a decision tree abstractly, I'm going to try to show what one is and how it is used as I construct a decision model.

21.2 THE DILEMMA-PROJECT COMETAIL

A balloon flight crew has been asked to consider flying a 4,500 pound payload to an altitude of 105,000 feet to study the tail of a comet. The payload can be flown on a mylar scrim balloon which will cost $60,000. The scrim balloon will weigh 2,000 lbs. The manager of the crew believes that a modern polyethylene balloon might also be satisfactory, although experience with polyethylene carrying such a heavy payload is quite limited. Such a balloon will weigh 1,500 lbs and cost $10,000. (The reader is warned that these numbers and all others used here are fictitious. They are used to demonstrate a method of solving a problem, and no conclusions about any actual situation should be drawn from them.)

If a flight with one balloon were in every way equivalent to a flight with the other, the crew would choose the polyethylene balloon because of the obvious cost saving. Several differences exist, however. The crew considers the scrim balloon to be almost certain to succeed. Therefore any failure which may occur if the scrim balloon is used is likely to be due to operational difficulties, that is electronics, hardware, rigging, launch, etc. The polyethylene balloon is believed to have a lower probability of withstanding the rigors of handling and flight than the scrim, provided that operational factors do not tip the scales one way or the other, but the crew has had more experience with polyethylene balloons, and it feels that the probability of encountering operational difficulties during inflation and launch is less with polyethylene than with scrim.

The scientist and his sponsor understand in a general way the problems of selection, and they know that a flight attempt on either balloon may fail. The time during which the comet may be viewed will permit the equipment to be flown twice during the comet's passage with reasonable opportunity for repair between flights. They are willing to consider a second flight if that seems appropriate. Therefore, the crew may also consider the possibility of using two or more balloons and making at least two flight attempts if necessary.

If no more information than this were available, the manager of the crew would face several possible alternate solutions and he would, through some means perhaps not even fully understood to him, make a decision. He can buy the scrim balloon, and the cost for the balloon and helium will be $67,150, assuming that helium costs a dollar per pound of lift and that he will inflate to 10 percent free lift. He can buy a polyethylene balloon, and the cost for balloon and helium will be $16,000. He can buy two polyethylene balloons and know that he can attempt two flights at a cost for the balloons and helium of $33,200. Such a course will involve additional time and expense on the part of the scientist and additional balloon flight crew time. He can buy three polyethylene balloons and attempt three flights at a cost for balloons and helium of $49,800. Perhaps he might even consider buying two scrim balloons or a scrim balloon and a polyethylene balloon.

It is clear to him that at the very least he needs to calculate more carefully the cost of flying two or more flights on polyethylene, but even if he finds that two such flights can be conducted at a cost comparable to one flight on scrim, should he select the polyethylene? Not necessarily! If the costs of two programs are equal, he should select the one having the better probability of success. This tells him that he also needs to incorporate some measure of reliability into his decision making process.

21.3 A SIMPLE DECISION TREE

As a first try at constructing a decision model, the manager drew Figure 21.1. Events which may occur during the ballooning operation are marked by rectangular boxes. In this tree the event is described briefly in each box, but each event is also identified by a number - just below the event box. Events follow in sequence from left to right along each branch of the tree. The final event along any branch is either success or failure, designated by S and F respectively.

The chart implies that once inflation is started (event 1) four events only may follow. These are: event 2 - inflation will be successful and the balloon will be

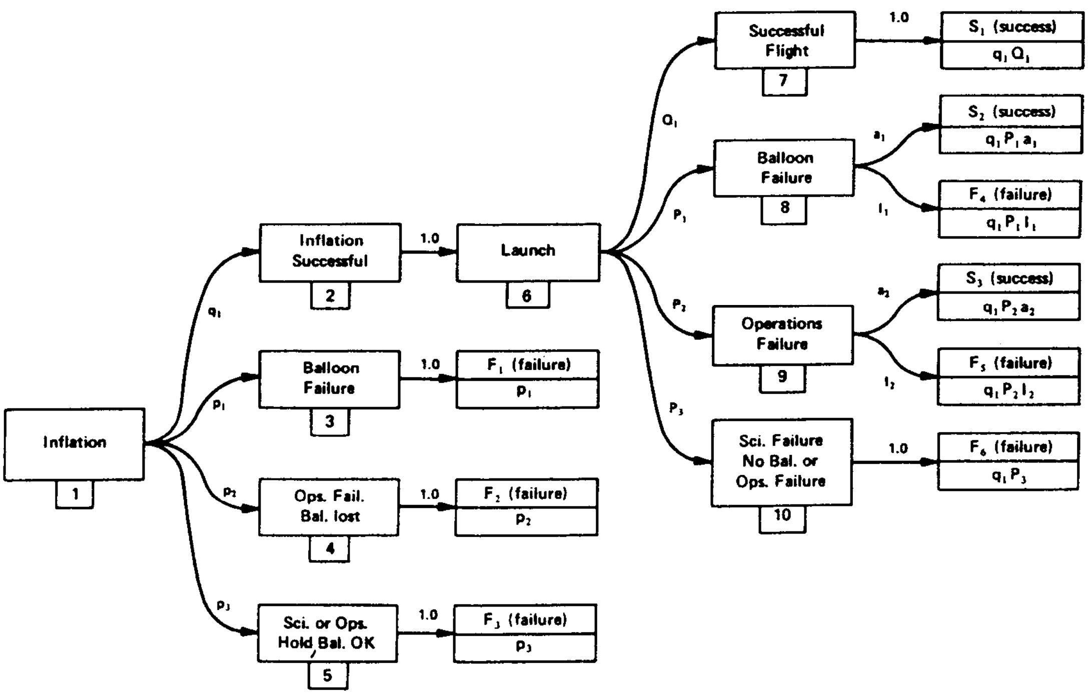

Figure 21.1. Balloon Flight Decision Tree for One Balloon and One Inflation

launched, event 3 - the balloon will fail in some manner during inflation and because the payload can not be flown, insufficient scientific data will be collected and the operation will fail, event 4 - an operations failure of some sort will occur which will result in loss of the balloon and lead to failure of the entire operation, and event 5 - a hold because of some problems on the part of the scientific or operations crew will result in expenditure of the helium but will leave the balloon usable.

Another manager might have chosen different events. The important thing is that the events should be pertinent, and they should be both mutually exclusive and exhaustive, that is they should all be recognizable, events such that if one occurs no other can occur, and taken all together they should include all possible events. One might ask, "what if inflation is started and stopped almost immediately so that essentially no gas is expended and the balloon is not damaged?" Under such conditions the manager would consider that event 5 had occurred, although the cost would not be the same as if all gas had been expended. Thus, somewhat arbitrary definitions are accepted in order to simplify the chart. They are akin to approximations often accepted in engineering practice. They should be accepted only with knowledge of their implications, however.

The probability that any one of the events will follow another event is given along the arrow joining the event boxes. For example, q_1 is the probability of a successful inflation leading to an attempted launch once inflation is started. The sum of the probabilities along a set of arrows proceeding from any event is 1. Thus

$$q_1 + p_1 + p_2 + p_3 = 1 \text{ and } Q_1 + P_1 + P_2 + P_3 = 1.$$

Note that along some arrows the probability is shown as 1.0, meaning that the next event is certain to follow. By definition, than, a successful inflation will be followed by an attempted launch. Also by definition - if launch is not attempted, sufficient scientific data will not be collected, and the mission will have failed.

Even if the launch is attempted, however, the mission can fail.The ways in which success or failure can occur following a launch attempt are shown following event 6 on the chart. They are: event 7 - a successful flight in which sufficient scientific data are collected, event 8 - failure of the balloon, but the failure may occur after the scientific mission is accomplished and so not preclude success of the mission, event 9 - an operations failure occurs, but again the mission may succeed in spite of it, and event 10 - the scientific equipment may malfunction or insufficient data may be obtained for reasons other than operations or balloon failure.

The probability that some combination of events will occur is calculated by following the usual rules for combining probabilities. The tree helps in keeping the procedure straight. Thus the probability that success will occur as a result of a particular sequence of events is obtained by taking the product of all the probabilities found along the branch representing that sequence of events. In Figure 21.1 the probability of success and failure at the end of each branch is shown in the lower part of the success and failure boxes. The overall probability of success may be obtained by summing the probabilities in all the boxes marked S, and the overall probability of failure may be obtained by either summing all the probabilities of failure or by subtracting the probability of success from 1.

The probability of success is

$$P_S = q_1 [Q_1 + P_1 a_1 + P_2 a_2] .$$

The probability of failure is

$$P_F = p_1 + p_2 + p_3 + q_1 [P_1 I_1 + P_2 I_2 + P_3] .$$

It may not be obvious that $P_S = 1 - P_F$, but if one makes use of the identities

$$1 - q_1 = p_1 + p_2 + p_3$$

$$1 - Q_1 = p_1 + p_2 + p_3$$

$$1 - a_1 = I_1$$

$$1 - a_2 = I_2$$

he can readily determine that it is true.

By using appropriate probability values for each of the balloons, the manager can now calculate the probability of success and failure and compare them with cost. Let's assume that the probabilities he needs are those given in Table 21.1. We'll discuss in the next section how he might have arrived at such values.

The probability of success with the scrim balloon ($P_{S,s}$) is

$$P_{S,s} = 0.93 [0.86 + 0.01 \times 0.25 + 0.07 \times 0.40]$$
$$\approx 0.83$$

and with the polyethylene

$$P_{S,p} = 0.89 [0.82 + 0.07 \times 0.10 + 0.05 \times 0.40]$$
$$\approx 0.75.$$

Table 21.1. Operational Probabilities

	Scrim	Poly-ethylene	Sequence of Events to Which Probability Applies
q_1	.93	.89	Successful inflation following start of inflation.
p_1	.01	.06	Balloon fails on inflation attempt.
p_2	.02	.03	Balloon is lost during inflation due to an operations failure.
p_3	.04	.02	Operations or Scientific hold causes loss of gas but not balloon.
Q_1	.86	.82	Successful flight following successful inflation.
P_1	.01	.07	Balloon fails during flight.
P_2	.07	.05	Operations failure occurs during flight.
P_3	.06	.06	Scientific failure during flight prevents success.
a_1	.25	.10	Balloon failed but adequate scientific data obtained.
I_1	.75	.90	Balloon failed and inadequate data obtained.
a_2	.40	.40	Operations failure but adequate data obtained.
I_2	.60	.60	Operations failure and inadequate data obtained.

The probabilities of failure with the scrim and polyethylene are respectively:

$$P_{F,s} \approx 1 - 0.83 = 0.17$$

$$P_{F,p} \approx 1 - 0.75 = 0.25.$$

The costs which the manager has estimated for the operation depicted by Figure 21.1 are shown in Table 21.2. From Table 21.2, the manager can assign costs to each success or failure event on the decision tree. Event 3 will cost the sum represented by the symbols G + E + B + H + I and so will event 4. Event 5 will cost G + E + H + I if the balloon is not damaged and its cost can be recovered. If the balloon is designed just for this flight, however, and none of its cost can be recovered, the cost of event 5 is the same as events 3 and 4. The costs to be associated with each of the successes or failures following launch are all the same, and they are obtained by summing cost categories G, E, B, H, I and F.

Table 21.2. Definition and Cost of Each Cost Category

Cost Symbol	Cost - Scrim	Cost - Poly-ethylene	Definition
G	$150,000	$150,000	Preparing and delivering scientific payload to the launch site, including all scientific crew costs prior to layout for inflation.
	3,000	3,000	Balloon crew costs for preparations leading up to layout for first attempted inflation.
E	1,000	1,000	Cost of expendable hardware, rigging, etc. used on each attempted flight.
B	60,000	10,000	Cost of one balloon.
H	7,150	6,600	Cost of helium for one inflation.
I	1,500	1,500	Balloon crew and scientific crew costs for layout, etc. through inflation.
F	2,000	2,000	Cost of launch and ensuing flight operation.
R	10,000	10,000	Cost of repairing scientific and flight equipment after a flight. Crew per diem and salaries are included.
*D	1,200	1,200	Cost of scientific crew per diem, salaries, etc. due to a delay resulting from an aborted inflation.

*Since such a delay will occur only if a balloon is available for another inflation attempt, this delay is principally a wait for a suitable day. It may be as little as 1 day or it may be many. The manager assumes 3 based on normal activity, climatology, etc.

The manager can now determine the risk of flying with polyethylene by multiplying the probability of each failure event by the cost of that event and summing all the products. He can do the same thing with scrim, and then he can compare the two risks. This is the simplest way of combining the measured of performance (the probabilities of failure and the costs) into a single measure of utility. If he accepts this measure as an adequate one, he would choose the balloon for which the risk is lower.

For balloons characterized by the data of Tables 21.1 and 21.2, the various probabilities of failure, the corresponding costs and the product of the two are shown in Table 21.3. The risk is the sum of the products. It is slightly lower with scrim.

Table 21.3. Computation of Risk for a Flight of One Balloon and One Inflation

Prob. Symbol	Scrim			Polyethylene		
	Prob. of Failure	Scrim Cost	Product	Prob. of Failure	Polyethylene Cost	Product
$q_1P_1I_1$	.007	$224,650	$ 1,573	.056	$174,100	$ 9,750
$q_1P_2I_2$	.039	"	8,761	.027	"	4,701
q_1P_3	.056	"	12,580	.053	"	9,227
p_1	.01	222,650	2,226	.06	172,100	10,326
p_2	.02	"	4,453	.03	"	5,163
p_3	.04	"	8,906	.02	"	3,442
Risk			$38,499			$42,609

Risk as a measure of utility does not take into account the value of the data one might get from a successful flight. In this case if the value of the data is expected to be less than the cost of the flight with a scrim balloon but greater than the cost with a polyethylene balloon, the scrim would clearly be the poorer choice. If this were true, however, to conduct the experiment with a balloon as a vehicle probably wouldn't be worthwhile anyway, and another alternative should be sought.

If the worth of a successful flight is known, a better measure of utility than risk can be used. Often termed expectation by game and decision theorists, it is the sum of all the products of the individual probabilities of success and their corresponding gains (worth of the experiment less the cost) less the risk. To illustrate for this example, let's assume the value of a successful experiment is $1,000,000. Then the gain if one succeeds is ($1,000,000 - $224,650) for scrim and ($1,000,000 - $174,100) for polyethylene. It is the same in this example regardless of whether one succeeds via event 7 or event 8 or event 9, because the cost is the same for each of these events. The expectation is then

$$\left[(q_1Q_1 + q_1P_1a_1 + q_1P_2a_2) \text{ (value of experiment - cost of flight) - risk}\right].$$

For scrim this is (0.828 x 775,350 - 38,499) or \$603,490. For polyethylene it is \$580,120. Expectation also favors scrim under these conditions.

The worth of an experiment is rarely known, a priori, but sometimes it is possible to say that it has a value greater than or less than some specified value. We can calculate a number from the information we have which may be useful as a discriminant between courses of action.

Let V = value of a successful flight, C_s and C_p be the costs of a flight with scrim and polyethylene respectively and r_s and r_p the risks with scrim and polyethylene. Then in this simple case

$$e_s = (V - C_s)\, P_{S,s} - r_s \text{ (expectation with scrim)}$$

and

$$e_p = (V - C_p)\, P_{S,p} - r_p \text{ (expectation with polyethylene).}$$

If one equates the expectation with scrim to that with polyethylene, he can calculate the value of V (designated by V_e) for which scrim and polyethylene are equally attractive. Thus

$$V_e = \frac{\left(C_s P_{S,s} + r_s\right) - \left(C_p P_{S,p} + r_p\right)}{P_{S,s} - P_{S,p}}$$

$$= \frac{(224,650 \times 0.828 + 38,499) - (174,100 \times 0.754 + 42,609)}{0.828 - 0.754}$$

$$\approx \$684,000. \qquad (21.1)$$

If the sponsor or the scientist regards the worth of a successful experiment to exceed this, the scrim should be chosen. If not, the polyethylene should be chosen unless the worth of the experiment doesn't greatly exceed the cost of a flight on polyethylene. If it doesn't, the advisability of flying the experiment on any balloon should be seriously questioned.

21.4 ESTIMATING PROBABILITIES

Normally a balloon flight facility manager doesn't think in terms of probabilities such as those listed in Table 21.1, but he does frequently know the percentage of flight attempts which result in successful flights. He also can usually determine from records available to him what fraction of the balloons fail during inflation and what fraction fail during flight. In these and similar records he may have the data required to form good estimates of some of the probabilities. q_1 in Table 21.1 may be estimated by dividing the total number of attempted inflations into the number of successful ones, taking care to define a successful inflation as it is defined in Table 21.1. An error could easily be made here by including as successes those inflations which were successful except that for operations or scientific reasons they were not followed by a launch attempt. Assume for purpose of illustration that 100 inflations of scrim balloons in the appropriate size and load carrying range have been attempted. Of these 93 were successful, one failed due to a defective balloon, two were destroyed due to operational errors and four were either partially or wholly inflated but for operational or scientific reasons were then deflated and stored for future use without damage to the balloon. Then the numbers 93/100, 1/100, 2/100 and 4/100 are estimates of the probabilities q_1, p_1, p_2 and p_3. Now a population of 100 is a fairly large one in ballooning if the balloon size and payload range are at all restrictive, and when 99 out of 100 balloons perform satisfactorily during inflation (there were three failures, but two were believed or known to have been caused by operational failures), one is willing to concede that the balloons perform reliably through the inflation operation. In fact, most of us probably wouldn't reject the statement that the reliability through inflation is 0.99.

Would we as readily accept the statement that the probability of failure is 0.01 knowing that only one failed? That failure could have been an accident, or perhaps it was an accident that only one failed. Several others may have been on the verge or failure. We can not doubt that the probability of failure is 0.01 without also doubting that the reliability is 0.99, however. The point is that even with a record of flights which is large in terms of scientific ballooning, it is clear that we question the accuracy of probability values calculated from it.

I have outlined a way to use past performance records to estimate probability values which can be used in decision models, but I have also cast doubt on the accuracy of the values obtained, especially when the sample size is small. Another reason exists for questioning the accuracy of probability estimates based on past performance. What if the company which laminates the mylar film to the dacron scrim has made a subtle change in the lamination process? Do the results of the past still provide a basis for making estimates about the behavior of scrim balloons in the future? We don't know, of course, but if we accept the hypothesis that the past offers no information about the future because of this change, we must start over again with material tests, test flights, etc., if we are to have any basis for decisions about courses of action for the future. Changes such as the one suggested here are rarely so drastic that they cause us to lose complete faith in the value of past performance as a measure of future performance. Usually we accept the thesis that past performance of a system provides an acceptable first approximation to future performance of a similar system; then we modify that first approximation to account for known or anticipated differences in the system.

Because we know that the accuracy of the best probabilities we can obtain to use in a formal decision model is open to question, we may question the wisdom of using them at all in decision making. But can we make a decision without using some measure of reliability? If we choose from two courses of action the one which will be less expensive if every aspect of the operation is successful, we are implicitly assuming that the difference in probability of success of the two systems is sufficiently small that the cost difference is more significant than reliability. If we are determined to buy the balloon system which is more likely to succeed, we must first decide which system that is, requiring at least that we made a comparative estimate of the likelihood of success. We are also implicitly assuming that the difference is great enough to compensate in some way for the difference in cost if the one we select is the more expensive one. It is difficult to conceive that a decision can be made between two balloon systems unless the decision maker does make either explicit or implicit assumptions about the relative likelihood (probability) of success of the systems. If such assumptions are indeed made, the decision maker should use the best information he has to estimate the likelihood of success, and he should use the estimates he makes in such a way that he understands at least qualitatively how they influence the decision.

If we accept these arguments and the conclusion, we see the need for consciously determining and using probabilities in decision making, and we recognize that probabilities which are applicable to ballooning will always be estimates. We recognize the need therefore to understand how these estimates affect our decisions so that we can take into account any doubts we may have about the accuracy of the estimates.

In the statement of the problem, it was pointed out that experience with flights of polyethylene balloons carrying heavy payloads is quite limited. Any estimate of the probability of success of a polyethylene balloon system of the type required which can be made from past performances of comparable systems will therefore be very doubtful. On the other hand, from his records the manager knows that there is a wealth of data on polyethylene systems carrying smaller payloads. He knows that several flights carrying payloads nearly as heavy as this one have succeeded and a few have failed. Further, recent improvements have been made in the seal strength of polyethylene balloons. He must try to use this and other information available to him to make the estimates he needs.

He can simply make an educated guess at the value of each of the probabilities he needs for the decision model, using his experience and knowledge to guide him and making use of the fact that the sum of the probabilities of each of several groups must be unity. He can ask others who have experience in ballooning to make such guesses also, after assuring himself that each understands the purpose of the guesses and has reviewed the pertinent information available to him. An open discussion of information by those who are going to contribute guesses can be helpful. It will expose each to the other's concepts of what information is pertinent. It is probably better for each contributor to make his guesses privately than publicly in a meeting, however.

The various guesses can be combined in any number of ways. If all contributors are equally knowledgeable and all are considered to be unbiased, an unweighted average of all guesses of a particular probability may be best. The average then provides the group's best estimate and the variability of the responses serves as a measure of the uncertainty. A weighted average has some potential advantages over a straight average, however, because it allows the values contributed by the more knowledgeable participants to be given more emphasis. If the manager intends to use a weighted average, he should decide before he has the responses in hand what weights he will assign to the various contributions. Otherwise he may bias the results unduly by what he himself believes.

Let's assume that the probabilities given in Table 21.1 for polyethylene are the result of averaging the contributions of five knowledgeable people. Let's also assume that the person making the estimates most favorable to polyethylene turned in the following values for the operation using a polyethylene balloon:

$$q_1 = 0.91 \qquad Q_1 = 0.85$$
$$p_1 = 0.04 \qquad P_1 = 0.04$$
$$p_2 = 0.03 \qquad P_2 = 0.05$$
$$p_3 = 0.02 \qquad P_3 = 0.06$$

If the other values in Table 21.1 are used with these, we find that the probability of success with polyethylene is

$$P_{S,p} = 0.91\ [0.85 + 0.04 \times 0.10 + 0.05 \times 0.40]$$
$$= 0.91 \times 0.874 \approx 0.795\ .$$

Thus the most optimistic set of probability values shows an overall probability of success of 0.795 while the adopted set showed 0.754. Since the cost of the operations are unchanged, the expectation for polyethylene, using this more optimistic probability and assuming the value of a successful flight to be $1,000,000 is

\$622,310. This exceeds the \$603,490 calculated for scrim. Also, a successful flight must have a value of \$1,540,000 in order that the expectation with scrim will equal that with polyethylene. This contrasts with a value of \$684,000 using the polyethylene probability data of Table 21.1. The risk for polyethylene using $P_{F,p} = 0.205$ is \$35,280 contrasted to \$38,499 for scrim. Polyethylene would be the more logical choice unless the value of a successful flight is known to exceed \$1,540,000.

These results show that with the costs and experiment value assumed in this case, the various measures of utility are fairly sensitive to changes in reliability. But if the value of a successful experiment were known to exceed \$1,540,000, neither the expectation nor the discriminant would have indicated a change of choice from that made with the averaged probability data. If this were true, the manager would have been fairly confident that scrim was a good choice. If he had questioned that the value of the experiment was as great as \$1,000,000, however, he might have wished to compare measures of utility using one of the more pessimistic sets of estimates of the reliability of polyethylene. In the end, he probably used the set he considered to be the most accurate, but he understood how variations in the reliability of polyethylene could affect the measures of utility he was using.

Even with the best possible probability estimates, a bad decision may be made; a decision based on intuitive feelings about reliability is as likely to result in a bad decision as a good one. Therefore, unless one can devise an acceptable decision model which is not sensitive to the probabilities of success or failure, he must make the best estimates of reliability he can. The manager in this case should strive to find better ways of estimating probabilities for future decision models. Perhaps he can find ways to combine the results of materials tests, seal strength tests, manufacturer's quality control checks, etc., into meaningful balloon success or failure probabilities. He must not ignore the need for better estimates of the probability of success and failure in the various facets of operations either. Above all, however, he should not let the difficulty of obtaining acceptable probabilities lead him to adopt a decision model which implicitly assumes values of probability of success and failure for a balloon or for any aspect of the operation, unless he is aware that he is making the assumption and understands its consequences.

21.5 MORE COMPLEX DECISION TREES

The manager, having used decision theory to study a very limited set of alternatives, had a better understanding of its limitations than he had before, but he also had acquired an appreciation for its potential value. He decided to construct a model to study a more realistic set of alternatives. He assumed that he will have two balloons available, that two attempted launches are permissible and that he can go through as many as three inflations if necessary.

As a first step he decided to extend the simple decision tree to include the possibility of a second inflation. The tree, showing events only by number, is presented in Figure 21.2. The branches of the tree going through events, 2, 3 and 4 are exactly like the tree shown in Figure 21.1 except that the success twigs at the end of the branch have been gathered together into one success bundle and the failure twigs also are formed into one failure bundle. The probability that events will lead to either of these bundles once inflation is started is given in the box representing the bundle.

Several simplifications of the tree are possible. For the purpose to be served here, events 3 and 4 can be combined. Also events 7 through 10 can be combined into two events, success and failure. The branch on the tree growing through event 5 following the first inflation is exactly like the tree itself without that branch, except that on the tree event 5 leads to a second event 1 while on the branch event 5 leads to failure. The branch growing through events 1, 2, 6, etc. to success and failure is a complex, re-occurring branch which for this application can readily be simplified to the one shown in Figure 21.3. Making use of these simplifications makes it possible to redraw Figure 21.2 as shown in Figure 21.4. The sequence of letters following each S and F event box will be explained in the next paragraph. Calculating the probability that events will lead to any one of the success or failure boxes is simple from this diagram.

The sequence of letters following each success and failure box are the letter symbols for the cost categories shown in Table 21.2. They are written in a special sequence to help avoid errors. Starting with the shortest sequence, that following events 3 and 4 after the first attempted inflation, the costs are the common costs which aren't repeated (G), the cost of expendable hardware (E) which must be replaced after each launch attempt, the cost of the balloon (B), the cost of the helium (H) and the cost of the inflation operation I. The cost categories which occur if the balloon is flown after the first inflation are shown after the S_1 and F_1 event boxes at the top of the chart. The sequence is exactly the same as the sequence described above except that the cost of the flight (F) is added. This F should not be confused with a failure symbol, but the usage is so different that no confusion is likely. Enough has been said that it should be fairly obvious if one starts at the left side of the chart and adds cost categories as events occur along any branch, he has the appropriate sequence when he reaches the end of the branch. The sequence after each success will end with an F; the sequence after each failure will end with either an I or an F.

By using the cost symbol shown after each failure event and Table 21.2, the total cost of all the events leading to that failure event can be readily calculated.

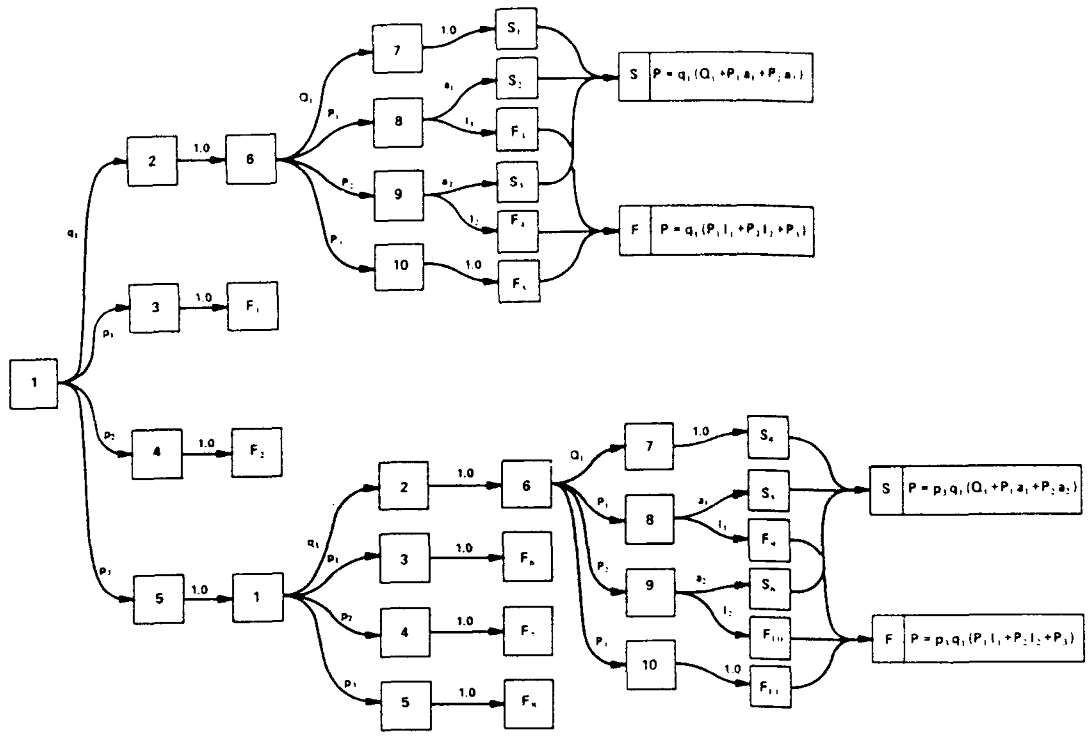

Figure 21. 2. Decision Tree for One Balloon and a Second Inflation if a Second One is Advantageous.

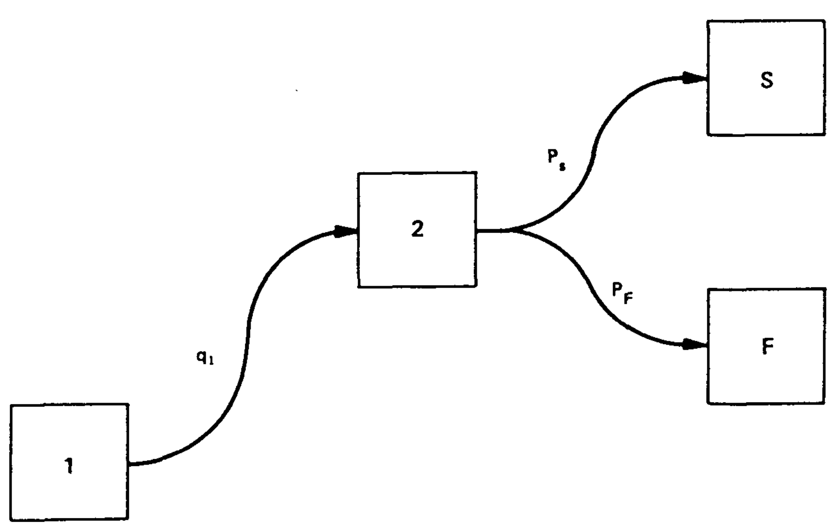

Figure 21. 3. Simplified, Recurring Branch in Which $P_S = (Q_1 + P_1a_1 + P_2a_2)$ and $P_F = (P_1I_1 + P_2I_2 + P_3)$. Also $P_S + P_F = 1$.

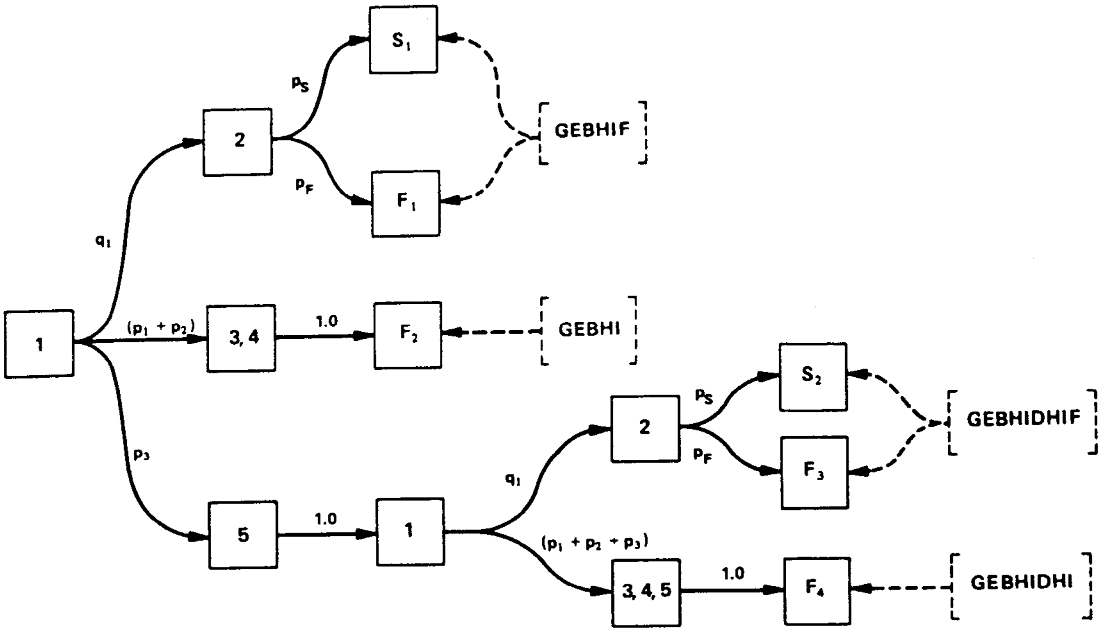

Figure 21.4. Simplified Decision Tree for One Balloon and as Many as Two Inflations. Note that $(p_1 + p_2 + p_3)$ along the lowest branch could be replaced by $(1 - q_1)$

That cost multiplied by the probability that failure will occur in that way gives the risk of failure through that particular chain of events, that is the events along that branch. As in the simple tree, the sum of all such risks is the risk of the entire project. Mathematically this can be written

$$r = \sum_{i=1}^{n} (C)_i \, (P_F)_i \tag{21.2}$$

where there are n failure events, $(P_F)_i$ is the probability of failure through the ith failure event and $(C)_i$ is the cost of all events leading up to and including that failure event.

Similarly, if the value of a successful experiment is known, the expectation can be written

$$e = \sum_{j=1}^{m} \left[V - (C)_j \right] (P_S)_j - \sum_{i=1}^{n} (P_F)_i \, (C)_i \tag{21.3}$$

where there are m success events.

Finally if one wishes to know the value the experiment must have in order that the polyethylene and scrim will have equal expectation, the following equation may be used:

$$V_e = \Big([ESC] - [EPC]\Big) \div \Big(P_{S,s} - P_{S,p}\Big) \tag{21.4}$$

in which

$$[ESC] = \sum_{j=1}^{m} \left(C_S\right)_j \left(P_{S,s}\right)_j + \sum_{i=1}^{n} \left(C_S\right)_i \left(P_{F,s}\right)_i$$

$$[EPC] = \sum_{j=1}^{m} \left(C_p\right)_j \left(P_{S,p}\right)_j + \sum_{i=1}^{n} \left(C_p\right)_i \left(P_{F,p}\right)_i$$

$$\left(P_{S,S} - P_{S,p}\right) = \sum_{j=1}^{m} \left(P_{S,s}\right)_j - \sum_{j=1}^{m} \left(P_{S,p}\right)_j \ .$$

Equations (21.2), (21.3) and (21.4) are general and may be used with decision trees of any complexity which are constructed like the ones used here. V_e is a sufficiently useful number that some additional discussion of it is worthwhile.

The terms in brackets in the numerator of Eq. (21.4) may be interpreted as expected costs of the project. The first as written above is a best estimate of the cost of the operation with scrim if only those balloons, inflations, services, etc., which are necessary to give success or lead to ultimate failure are used. One might view it as the best estimate we can make of the average cost of a large number of similar projects carried out with scrim. Many will succeed with one balloon, one inflation and one flight attempt. These will not be very expensive. A few will use two balloons, two inflations, etc., and cost more.

The difference between the two terms in brackets in the numerator is then a best estimate of the difference in cost of carrying out the experiment using scrim and polyethylene. The terms in the denominator are the overall probabilities of success with scrim and polyethylene in that order.

V_e may take any value, being undefined at plus and minus infinity. If $V_e > 0$, the more costly operation is also the one more likely to succeed. A decision between them can only be made if the actual value of the experiment is deemed to be greater than or less than V_e. If it is greater than V_e, one chooses the more expensive alternative. If it is less, the lower cost alternative is preferable. If $V_e = 0$, there is no difference in the costs, but there is a difference in the probability of success, and the more reliable alternative should be chosen. If $V_e < 0$, the more costly operation is the one less likely to succeed. The choice is obvious. Finally, V_e may be undefined (that is $\pm \infty$) because the probability of success with one balloon system is equal to the probability of success with the other. The lower cost system should then be chosen.

The simplification in tree construction leading to Figure (21.4) makes trees with additional alternatives appear tractible. Consequently the manager proceeded to construct a tree which would let him compare the alternatives open to him if he has two balloons and the opportunity for as many as three inflations and two launches. Figure (21.5) is the decision tree which resulted.

The events which are numbered are defined exactly as they were in the first simple tree. The success and failure events have also been identified by number as well as by the letters S and F. The probability symbols shown along the arrows between events have two numbered subscripts instead of a single one as before. The first subscript is used exactly as before. The second one identifies the balloon being used. If the second balloon were exactly like the first, this would not be necessary; but if one wishes to consider the possibility of using a scrim balloon with a polyethylene backup, the probabilities along the branches must be differentiated.

Note that some event boxes have two or more numbers in them. This was done because it was not helpful to identify the events separately for the decision the manager was trying to make. In the selection of a balloon, for example, it is not important whether a balloon fails because of defects or because it is damaged during the operation; therefore events 3 and 4 can be combined into an event identified as 3, 4 and defined as the occurrence of either event 3 or event 4.

The scientist's sponsor might use a similar decision tree to help him select the best combination of crew and balloons. He would be interested in differentiating crew performance and cost as well as balloon performance and cost, and so might want to keep events 3 and 4 separated; he might wish to consider other events as well.

When events are combined as 3 and 4 are here, the probability of occurrence of the combined event is the sum of the probabilities of the individual events since the individual events are mutually exclusive. If combining leaves the possibility of only two events following a given event, the probabilities are easier to show as the probability of the occurrence of one of the events and the probability of non-occurrence of that same event. This is done following the third inflation where $q_{1,2}$ and $(1 - q_{1,2})$ are used instead of $q_{1,2}$ and $(p_{1,2} + p_{2,2} + p_{3,2})$. This is possible here because when all permissible inflation attempts have been made, event 5 also results in failure even though a balloon may still be available.

Finally, note that some of the letters symbolizing cost categories are underlined. Those which are not underlined apply to the first balloon used in the

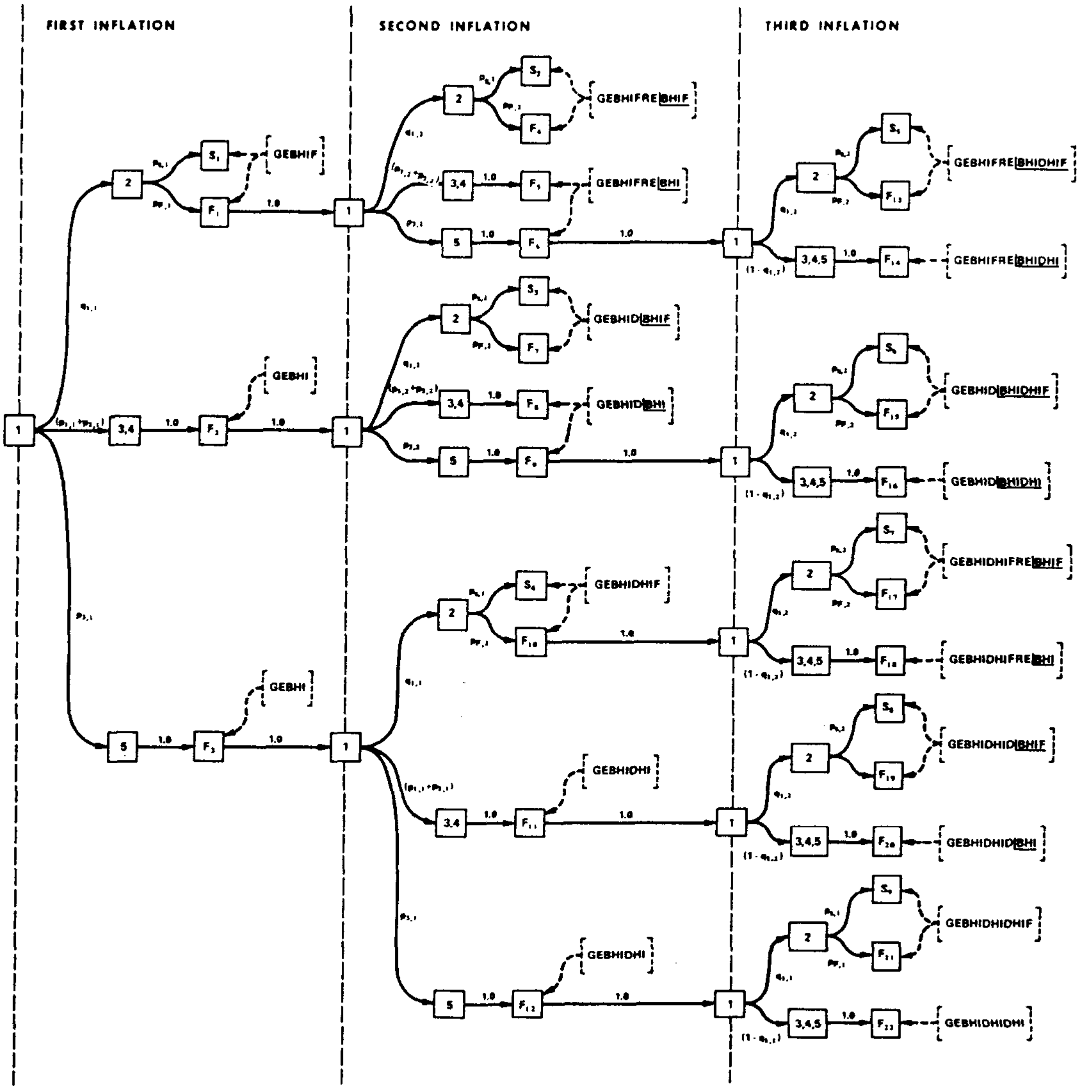

Figure 21.5. Decision Tree for an Operation Involving Up to Two Balloons, Three Inflations and Two Launches. If both balloons are identical, $q_{1,1} = q_{1,2}$; $p_{S,1} = p_{S,2}$, etc. If the balloons are different, in general $q_{1,1} \neq q_{1,2}$; etc.

operation; those which are underlined apply to the second. This becomes important only if the costs are different, for example if the first balloon is scrim and the second is polyethylene.

From Figure 21.5 and Table 21.4, probabilities can now be readily combined to determine the probability of each of the success or failure events. From Figure 21.5 and Table 21.2, the cost of each success or failure event can be determined. These results are given in Table 21.5.

Table 21.5 is essentially a computation form from which the value of the various terms of Eqs. (21.2) and (21.4) can be determined.

Table 21.4. Probability of Succession of Events for the Decision Tree Depicted by Figure 21.5

Succession of Events	Probability Symbol	Scrim	Poly-ethylene
1 to 2	$q_{1,1}$ or $q_{1,2}$	.93	.89
1 to 3	$p_{1,1}$ or $p_{1,2}$	.01	.06
1 to 4	$p_{2,1}$ or $p_{2,2}$	.02	.03
1 to 5	$p_{3,1}$ or $p_{3,2}$	.04	.02
2 to S	$p_{S,1}$ or $p_{S,2}$	.890	.847
2 to F	$p_{F,1}$ or $p_{F,2}$	.110	.153
1 to 3, 4	$(p_{1,1} + p_{2,1})$ or $(p_{1,2} + p_{2,2})$	.03	.09
1 to 3, 4, 5	$(1 - q_{1,1})$ or $(1 - q_{1,2})$	.07	.11

Table 21.6, derived from Table 21.5, gives the overall probability of success and failure, the risk $\left(\Sigma\,(C_i)\,(P_F)_i\right)$ and the expected cost $\left(\Sigma\,(C)_j\,(P_S)_j + \Sigma\,(C)_i\,(P_F)_i\right)$ of a number of possible alternatives. The second column of Table 21.6 shows which failure probabilities in Table 21.5 to add to obtain the overall probability of failure of the mission (shown in column 3) for the alternative stated in column 1. The probabilities given in columns 3 and 5 are carried to the third place to the right of the decimal, as they were in Table 21.5, for consistency in computations only; they aren't known that accurately.

Column 4 of Table 21.6 shows which success probabilities in Table 21.5 to sum to obtain the overall probability of success, and column 5 gives the numerical

Table 21.5. Summary of Probabilities and Costs and Their Products for Various Balloon Combinations. Probability and cost symbols are taken from Figure 21.5. Costs are based on Table 21.2, probabilities on Table 21.4. The column heading "scrim/polyethylene" indicates that the scrim balloon was used first.

S or F	Probability	Cost	Probability				Cost				Cost X Probability			
Event	Symbol	Symbol	Scrim/ Scrim	Scrim /Poly	Poly/ Scrim	Poly/ Poly	Scrim/ Scrim	Scrim /Poly	Poly/ Scrim	Poly/ Poly	Scrim/ Scrim	Scrim /Poly	Poly/ Scrim	Poly/ Poly
S_1	$q_{1,1}P_{S_1}$	*CF	.828	.828	.754	.754	$224,650	$224,650	$174,100	$174,100	$186,010	$186,010	$131,271	$131,271
S_2	$q_{1,1}P_{F,1}q_{1,2}P_{S,2}$	CFRE/BHIF	.085	.077	.113	.103	306,300	255,750	255,750	205,200	26,036	19,692	28,900	21,136
S_3	$(p_{1,1}+p_{2,1})q_{1,2}P_{S,2}$	CD/BHIF	.025	.023	.074	.068	294,500	243,950	243,950	193,400	7,362	5,610	18,052	13,151
S_4	$p_{3,1}q_{1,1}P_{S,1}$	CDHIF	.033	.033	.015	.015	234,500	234,500	183,400	183,400	7,738	7,738	2,751	2,751
S_5	$q_{1,1}P_{F,1}p_{3,2}q_{1,2}P_{S,2}$	CFRE/BHIDHIF	.003	.002	.004	.002	316,150	265,050	265,600	214,500	948	530	1,062	429
S_6	$(p_{1,1}+p_{2,1})p_{3,2}q_{1,2}P_{S,2}$	CD/BHIDHIF	.001	.000	.003	.001	304,350	253,250	253,800	202,700	304	---	761	203
S_7	$p_{3,1}q_{1,1}P_{E_1}q_{1,2}P_{S,2}$	CDHIFRE/BHIF	.003	.003	.002	.002	316,150	265,600	265,050	214,500	948	797	530	429
S_8	$p_{3,1}(p_{1,1}+p_{2,1})q_{1,2}P_{S,2}$	CDHID/BHIF	.001	.001	.001	.001	304,350	253,800	253,250	202,700	304	254	253	203
S_9	$p_{3,1}p_{3,1}q_{1,1}P_{S,1}$	CDHIDHIF	.001	.001	.000	.000	244,350	244,350	192,700	192,700	244	244	---	---
F_1	$q_{1,1}P_{F,1}$	CF	.102	102	136	.136	224,650	224,650	174,100	174,100	22,914	22,914	23,678	23,678
F_2	$p_{1,1}+p_{2,1}$	C	.030	.030	.090	.090	222,650	222,650	172,100	172,100	6,680	6,680	15,489	15,489
F_3	$p_{3,1}$	C	.040	.040	.020	.020	222,650	222,650	172,100	172,100	8,906	8,906	3,442	3,442
F_4	$q_{1,1}P_{F,1}q_{1,2}P_{F,2}$	CFRE/BHIF	.010	.014	.014	.019	306,300	255,750	255,750	205,200	3,063	3,580	3,580	3,899
F_5	$q_{1,1}P_{F,1}(p_{1,2}+p_{2,2})$	CFRE/BHI	.003	.009	.004	.012	304,300	253,750	253,750	203,200	913	2,284	1,015	2,438
F_6	$q_{1,1}P_{F,1}p_{3,2}$	CFRE/BHI	.004	.002	.005	.003	304,300	253,750	253,750	203,200	1,217	508	1,269	609
F_7	$(p_{1,1}+p_{2,1})q_{1,2}P_{F,2}$	CD/BHIF	.003	.004	.009	.012	294,500	243,950	243,950	193,400	884	976	2,196	2,320
F_8	$(p_{1,1}+p_{2,1})(p_{1,2}+p_{2,2})$	CD/BHI	.001	.003	.003	.008	292,500	241,950	241,950	191,400	292	726	726	1,531
F_9	$(p_{1,1}+p_{2,1})p_{3,2}$	CD/BHI	.001	.001	.004	.002	292,500	241,950	241,950	191,400	292	242	968	383
F_{10}	$p_{3,1}q_{1,1}P_{F,1}$	CDHIF	.004	.004	.003	.003	234,500	234,500	183,400	183,400	938	938	550	550
F_{11}	$p_{3,1}(p_{1,1}+p_{2,1})$	CDHI	.001	.001	.002	.002	232,500	232,500	181,400	181,400	232	232	363	363
F_{12}	$p_{3,1}p_{3,1}$	CDHI	.002	.002	.000	.000	232,500	232,500	181,400	181,400	465	465	---	---
F_{13}	$q_{1,1}P_{F,1}p_{3,2}q_{1,2}P_{F,2}$	CFRE/BHIDHIF	.000	.000	.000	.000	---	---	---	---	---	---	---	---
F_{14}	$q_{1,1}P_{F,1}p_{3,2}(1-q_{1,2})$	CFRE/BHIDHI	.000	.000	.000	.000	---	---	---	---	---	---	---	---
F_{15}	$(p_{1,1}+p_{2,1})p_{3,2}q_{1,2}P_{F,2}$	CD/BHIDHIF	.000	.000	.000	.000	---	---	---	---	---	---	---	---
F_{16}	$(p_{1,1}+p_{2,1})p_{3,2}(1-q_{1,2})$	CD/BHIDHI	.000	.000	.000	.000	---	---	---	---	---	---	---	---
F_{17}	$p_{3,1}q_{1,1}P_{F,1}q_{1,2}P_{F,2}$	CDHIFRE/BHIF	.000	.001	.000	.000	---	265,600	---	---	---	266	---	---
F_{18}	$p_{3,1}q_{1,1}P_{F,1}(1-q_{1,2})$	CDHIFRE/BHI	.000	.000	.000	.000	---	---	---	---	---	---	---	---
F_{19}	$p_{3,1}(p_{1,1}+p_{2,1})q_{1,2}P_{F,2}$	CDHID/BHIF	.000	.000	.000	.000	---	---	---	---	---	---	---	---
F_{20}	$p_{3,1}(p_{1,1}+p_{2,1})(1-q_{1,2})$	CDHID/BHI	.000	.000	.000	.000	---	---	---	---	---	---	---	---
F_{21}	$p_{3,1}p_{3,1}q_{1,1}P_{F,1}$	CDHIDHIF	.000	.000	.000	.000	---	---	---	---	---	---	---	---
F_{22}	$p_{3,1}p_{3,1}(1-q_{1,1})$	CDHIDHI	.000	.000	.000	.000	---	---	---	---	---	---	---	---

*C is used here as a substitute for the collection of cost symbols GEBHI which occur with every success or failure event.

Table 21.6. Probability of Success and Failure, Expected Cost, Risk and Maximum Cost and Risk for Several Possible Alternatives

Alternatives	Failure Symbols	Prob. of Failure	Success Symbols	Prob. of Success	$\Sigma(C)_j(P_S)_j$	(Risk) $\Sigma(C)_i(P_F)_i$	Expected Cost	Max Cost	Max Risk
2S, 3I	(F_4, F_5, F_7, F_8 & F_{13}-F_{20})	.020	(S_1-S_9)	.980	$229,894	$ 5,152	$235,046	$316,150	$ 6,323
1S, 1P, 3I	"	.031	"	.971	220,875	7,832	228,707	265,600	8,234
1P, 1S, 3I	"	.030	"	.969	183,580	7,517	191,097	265,600	7,968
2P, 3I	"	.054	"	.946	169,573	10,188	179,761	214,500	11,583
2S, 2I	(F_4-F_{12})	.029	(S_1-S_4)	.971	227,146	8,296	235,442	306,300	8,883
1S, 1P, 2I	"	.040	"	.960	219,050	9,951	229,001	255,750	10,230
1P, 1S, 2I	"	.044	"	.956	180,974	10,667	191,641	255,750	12,530
2P, 2I	"	.059	"	.941	168,309	12,093	180,402	205,200	12,107
1S, 2I	(F_1, F_2, F_{10}-F_{12})	.139	(S_1, S_4)	.861	193,748	31,229	224,977	234,500	32,596
1P, 2I	"	.231	"	.769	134,022	40,030	174,050	183,400	42,182
1S, 1I	(F_1, F_2, F_3)	.172	(S_1)	.828	186,010	38,500	224,510	224,650	38,640
1P, 1I	"	.246	"	.754	131,271	42,559	173,830	174,100	42,829
Column No. (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

probability of success. Column 6 is the sum of the products of the individual probabilities of success and the corresponding costs given in Table 21.5 and column 7 is a sum of the products of the individual failures and the corresponding costs. The numbers in column 7 are the mathematical risks of the various alternatives. The sum of corresponding numbers in columns 6 and 7 is entered in column 8, and it is the average cost of the alternative shown on that row in column 1.

Finally the last two columns are the maximum cost and the maximum risk respectively. The maximum risk being defined as the maximum cost times the overall probability of failure. Note that the numbers in the last column are slightly larger than the numbers in column 7. Also the average cost of each of the alternatives is less than its maximum cost.

It is interesting to note that for any of the alternatives which include one scrim balloon and one polyethylene balloon, the overall probability of success is slightly higher if the scrim is flown first, but the average cost is greater if they are flown in that order.

By combining the data from any two alternatives through the use of Eq. (21.4), the value of the discriminant V_e can be determined, and the alternatives can be compared. Tables 21.7 and 21.8 show such comparisons between each pair of alternatives.

Tables 21.7 and 21.8 are very similar. Both are special matrices. The rows and columns are identified by the maximum number of balloons and inflations which may be used in a particular course of action, and the order in which the balloons are given is the order in which they will be used. Therefore, 1S, 1P, 3I is different from 1P, 1S, 3I. The elements of the matrix, except the diagonal elements, are values of the discriminant which one obtains by comparing the two courses of action identified by the row and the column. The courses of action are arranged in order of cost with the most expensive in the top row and left column. This arrangement permits the discriminant values to be entered in the matrix in such a way that if the value of a successful experiment exceeds the matrix element, the course of action identified with the row on which the element is found is preferable to the course of action on the column. This rule is stated in the upper left corner of the matrix. Since the elements along the diagonal are not used for discriminant values, the cost and probability of success of the operation indicated by both the row and column are shown there for convenient reference.

Let's assume that the value of a successful experiment has been determined to lie between 3 and 3.5 million dollars. Since the most reliable choice is 2S, 3I, the manager selects the row labeled 2S, 3I and scans it, seeking a number larger than 3,000. This represents a discriminant value of $3,000,000 because all numbers in the body of the table are in thousands of dollars. He finds the number 3,995. Since the value V of the experiment is less than 3,995, the column alternative

Table 21.7. Comparisons of Various Courses of Action Using Expected Costs. Discriminant values V_e in the table are given in thousands of dollars.

Choose if $V<V_e$ / Choose if $V>V_e$	2S 2I	2S 3I	1S, 1P 2I	1S, 1P 3I	1S 2I	1S 1I	1P, 1S 2I	1P, 1S, 3I	2P 2I	2P 3I	1P 2I	1P 1I
2S, 2I	235,442 .97		586	∞	95	76	2, 920	22,172	1,834	2,227	304	284
2S, 3I	-44	235,046 .98	302	704	85	69	1, 809	3,995	1,401	1,626	289	271
1S, 1P, 2I			299,001 .96		41	34	9, 340		2,558	3,517	288	268
1S, 1P, 3I			-27	228,707 .97	34	29	2, 471	18,805	1,610	1, 958	271	252
1S, 2I					224,977 .86	14					554	478
1S, 1I						224,510 .83					855	685
1P, 1S, 2I					-351	-257	191,641 .96		749	1.188	94	88
1P, 1S, 3I			-4,211		-314	-237	-42	191,097 .97	382	493	85	80
2P, 2I					-557	-390			180,402 .94		37	35
2P, 3I					-532	-379			-128	179,761 .95	32	31
1P, 2I											174,050 .77	15
1P, I												173,830 .75

Table 21.8. Comparisons of the Various Courses of Action Using Maximum Costs. Discriminant values V_e are given in thousands of dollars.

Choose if $V<V_e$ / Choose if $V>V_e$	2S 3I	2S 2I	1S, 1P 3I	1S, 1P 2I	1S 2I	1S 1I	2P 3I	2P 2I	1P 2I	1P 1I
2S, 3I	$\boxed{316{,}150 \; .98}$	1,094	5,106	3,179	686	602	2,990	2,845	629	629
2S, 2I		$\boxed{306{,}300 \; .97}$	∞	5,055	653	3,571	3,672	3,370	608	609
1S, 1P, 3I			$\boxed{265{,}600 \; .97}$	985	283	290	2,222	2,157	411	426
1S, 1P, 2I				$\boxed{255{,}750 \; .96}$	213	243	4,125	3,370	386	404
1S, 2I					$\boxed{234{,}500 \; .86}$	298			555	564
1S, 1I						$\boxed{224{,}650 \; .83}$			699	683
2P, 3I					-235	-86	$\boxed{214{,}500 \; .95}$	1,860	175	210
2P, 2I					-366	-172		$\boxed{205{,}200 \; .94}$	127	166
1P, 2I									$\boxed{183{,}400 \; .77}$	620
1P, 1I										$\boxed{174{,}100 \; .75}$

(1P, 1S, 3I) is preferable to the row alternative according to the instructions in the upper left corner of the table. In effect we are saying that the operation using two scrim balloons and up to three inflations will not on the average be enough more reliable than an operation with one polyethylene balloon, one scrim balloon and up to three inflations to warrant the difference in expected cost if the value of the experiment is less than $3, 995, 000. Now the manager enters the 1P, 1S, 3I row and seeks a number larger than 3, 000. Since he finds none, he accepts the one polyethylene, one scrim, three inflation alternative as the best choice. He finds the expected cost ($191, 097) and the probability of success (0. 97) along the diagonal where the 1P, 1S, 3I row intersects the 1P, 1S, 3I column.

Using the procedure just used, we can construct a set of rules for choosing alternatives in this case. These are shown in Table 21. 9 for both the expected cost and maximum cost alternatives. The choices are ranked in the order of descending experiment value. If the value of the experiment is high enough, three scrim balloons and up to three inflations are preferred regardless of whether the charges are to be made only for the balloons, equipment and services actually used or for the whole program. The preferred program at the next lower experiment value includes one scrim and one polyethylene balloon and up to three inflations, but the order in which the balloons should be used differs between the two costing systems. If the charges are made only for those services, etc., actually used, the polyethylene balloon should be used first because the expected cost is lower proportionally than the slightly lower reliability. If all balloons, services, etc., are to be paid for, however, the alternative having the higher reliability is preferred. Having only one balloon and one inflation is never a good choice, although it is listed in Table 21. 9. It is the best choice only when the value of the experiment is less than the cost, and under such circumstances the experiment isn't worth conducting in this manner.

In discussing the one balloon, one inflation model, risk was considered as a measure of utility. From Table 21. 6, we can see that the alternatives would be ranked as follows by risk: (1) 2S, 3I; (2) 1P, 1S, 3I; (3) 1S, 1P, 3I; (4) 2S, 2I; and (5) 1S, 1P, 2I. The first two choices, if one were to use risk, correspond to the preferred choices indicated in Table 21. 9 for highly valued experiments. In uses similar to this, choices made using risk as a measure of utility have been found to agree generally with choices made using expectation when the value of the experiment is high. Risk is not a good measure of utility when the value of the experiment is low, however.

Now, what appears to be a good decision from the point of view of the manager may not be an acceptable solution to the sponsor. For example, the total amount of money available to the sponsor may be a deciding factor in eliminating some alternatives. Perhaps if he can hold his expenditure on Project Cometail to

Table 21.9. Rules for Choosing Alternative Courses of Action Under Two Systems of Charging

Charges Made for Only Those Balloons, Etc. Used				Charges Made for Maximum Program			
Experiment Value*	Choose	Expected Cost*	Prob. of Success	Experiment Value*	Choose	Cost*	Prob. of Success
> 3,995	2S, 3I	235	.98	> 5,106	2S, 3I	316	.98
3,995-493	1P, 1S, 3I	191	.97	5,106-2,222	1S, 1P, 3I	266	.97
493-32	2P, 3I	180	.95	2,222-1,860	2P, 3I	214	.95
32-15	1P, 2I	174.1	.77	1,860-166	2P, 2I	205	.94
< 15	1P, 1I	173.9	.75	< 166	1P, 1I	174	.75

*Thousands of dollars.

\$225,000 or less, he will have money to sponsor another potentially valuable experiment. He can use the information in Table 21.9 as input to his own decision model and weigh these alternatives against others open to him. If he should decide that \$225,000 is all he can invest in this experiment, two polyethylene balloons and three inflations is the preferred alternative.

On the other hand, suppose the sponsor is trying to decide between balloons and a satellite as vehicles for Project Cometail. He believes that for \$5,000,000 the probability of acquiring sufficient acceptable date by satellite is virtually certain, say 0.999. Furthermore, he is willing to invest the \$5,000,000 for this purpose unless an acceptable alternative can be found. The manager can now accept the fact that the data which can be acquired by successfully carrying out the experiment are worth at least \$5,000,000. Using that value for V, he would choose 2S, 3I or 1S, 1P, 3I depending on the mode of charging.

Once again, if the sponsor is trying to decide between balloons and a satellite, he will note that there is still a 0.02 probability of failure with the best alternative offered here using balloons as vehicles. If the satellite offers a 0.999 probability of success, he can calculate that the value of the Cometail data must equal approximately \$250,000,000 for the expectation from the satellite to equal the expectation from the best balloon choice. This approximation is made using Eq. (21.4) as follows:

$$V_e = \frac{5,000,000 - 316,150}{0.999 - 0.98} = \$246,518,210$$

This is strictly valid only if the total cost of each program is independent of the success or failure of the program [that is, in Eq. (21.4), $(C)_j = (C)_i$]. That the value of a successful experiment will be so high seems unlikely, but still the sponsor may not be particularly happy with a 0.02 probability of failure.

The time during which the comet can be viewed satisfactorily does not permit more than two flights by one crew, but perhaps a parallel effort by two crews offers an answer. Each effort can be independent of the other; therefore, if the crews are equally competent, the probability of success with the second crew should equal that with the first. Also if they are equally efficient, the costs should be equal.

The maximum cost of such a double effort with two scrim balloons and three inflations each would then be \$632,300 and the probability of success, $P_{S,D}$, of the dual effort is the probability that at least one effort will succeed. It may be calculated in either of the following ways:

$$P_{S,D} = P_{S,1} + P_{S,2} - P_{S,1}P_{S,2}$$

or

$$P_{S,D} = \left(1 - P_{f,D}\right) = \left(1 - P_{f,1}P_{f,2}\right)$$

where $P_{S,1}$ is probability of success of one effort.

$P_{S,2}$ is probability of success of the other effort, and $P_{f,1}$ and $P_{f,2}$ are corresponding probabilities of failure. With the choice of two scrim balloons and three inflations,

$$P_{S,D} = 0.98 + 0.98 - 0.98 \times 0.98 = 0.9996$$

or

$$P_{S,D} = 1 - 0.02 \times 0.02 = 0.9996 .$$

This suggests a probability of success which exceeds that assumed for the satellite at less than one eighth the cost. Dual efforts with one scrim, one polyethylene and three inflations would provide a probability of success comparable with that of the satellite at one ninth the cost.

21.6 LOOKING BACK

Having started with a simple decision model which enabled him to make quantitative comparisons between the utility of a single scrim balloon with one inflation and that of a single polyethylene balloon with one inflation, the manager then developed a more complex model which let him compare the utility of a number of different balloon operations with each other. From that model he developed quantitative data which were useful in making rough comparisons of the various balloon combinations with a satellite system. He realized, however, that his model for decision making was not designed to include the satellite; therefore, if he should wish to make a rigorous comparison with the satellite, he should review the decision model carefully and modify it as necessary to assure that just comparisons can be made. He also assumed that the results of the model were applicable to another crew without change, and he further assumed that the probability of failure of the two were independent. If the results deriving from these assumptions are to be considered seriously in making a decision, they should be carefully examined for validity.

This attempt by the manager to construct a realistic decision model has caused him to look systematically at what he believes to be the most significant aspects of the success or failure of a balloon flight operation. He may decide that he can construct a more realistic model for this particular problem; if so, he should do so. Whether he does so or not, he will realize that he needs to institute ways of obtaining better data (for example cost estimates, probabilities, etc.) for use in future models.

As a result of going through this exercise, he has a better appreciation of the way he should interact with the scientist, the scientist's sponsor, the balloon manufacturer, etc., if all pertinent facets of a problem are to be considered in proper perspective.

Ultimately, of course, a man makes the decision, but he should make it from the vantage point of the most careful possible consideration of all pertinent facts. Modern decision theory provides a systematic way of approaching that vantage point. How closely the decision maker comes to it depends on his skill and dedication.

References

Archibald, R.D., and Villoria, R.L. (1967) Decision Trees for Decision-making, Network-Based Management Systems (PERT/CPM), John Wiley and Sons, Inc.

Eilon, S. (1969) What is a decision, Journal, Institute of Management Science, **16**: (No. 4).

Fishburn, P.C. (1964) Decisions and Value Theory, Wiley, New York.

Sisson, R.L., Sieber, H.F., and Nagin, R.P. (1967a) Decision Making Revisited, Management Science Selections, Rev. Ed., Data Processing, Inc., Los Angeles, Calif.

Sisson, R.L., Sieber, H.F., and Nagin, R.P. (1967b) Bayesian Decision Theory, Management Science Selections, Rev. Ed., Data Processing, Inc., Los Angeles, Calif.

Tribus, M. (1969) Rational Descriptions, Decisions and Designs, Pergamon Press, New York and London.

Wheeler, R.E., and Peepels, W.D. Jr. (1969) Modern Mathematics for Business Students, Brooks/Cole Publ. Co., A Division of Wadsworth Publ. Co., Belmont, Calif.